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New descriptions of phase/shape transition regions in nuclei

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Abstract. Phase transitional behavior, critical-point symmetries, and the low-energy nuclear phase diagram are discussed, with emphasis on the behavior of simple spectroscopic observables. A perspective on the low-energy nuclear phase diagram from the standpoint of Landau theory is presented as well.

PACS. 21.60.-n Nuclear structure models and methods

1 Introduction

Recent discoveries [1–3] in the γ-ray spectroscopy of ^{152}Sm and neighboring nuclei have revealed a behavior that can be described in terms of phase transitions and phase coexistence. This has led to the development [4,5] of new "critical-point" symmetries to describe nuclear systems at these transition points. Spectra closely resembling these new paradigms were immediately recognized [6,7] in ^{152}Sm [$X(5)$] and ^{134}Ba [$E(5)$], and, soon thereafter in other nuclei in the $A = 150$ and 100 regions [8–11]. The upshot of this work is the addition of two analytic, parameter-free (except for scale) paradigms to the existing arsenal accessible to describe the low-energy nuclear structure. These benchmarks are illustrated in fig. 1. It is the purpose of this paper to briefly discuss the idea of phase transitions in nuclei, the discovery of $X(5)$, and a new approach to the discussion of nuclear equilibrium phases in terms of the classical Landau theory. We will finish with comments, derived from the Landau discussion, as to the nature of structural evolution in nuclei and the pervasiveness of the phase transitional behavior.

To begin we recall the ideas behind first- and second-order phase transitions in fig. 2. In a second-order phase transition the system energy $E(\beta)$ evolves as shown on the right side of the figure. For any given nucleon number (or set of model parameters driving the structural evolution) there is only one equilibrium configuration. With the addition of valence nucleons, the collectivity gradually grows and deformation develops (compare lines 1 and 2) in a continuous way.

In contrast, in a first-order phase transition, there are coexisting configurations —for example, one spherical, one deformed— and their relative energies vary as a function of nucleon number as shown on the left. The $Z = 64$ region illustrates the underlying microscopic basis [12] for

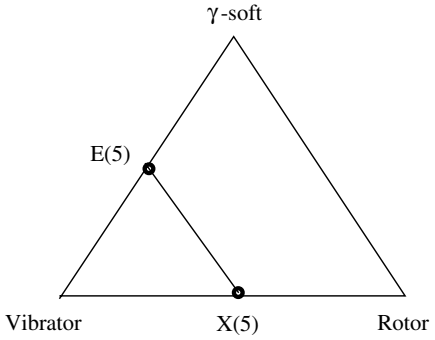

Fig. 1. Symmetry triangle for nuclear structure showing the three traditional limits of structure and the new critical-point paradigms $E(5)$ and $X(5)$. The line connecting them is a line of first-order phase transitions, terminating in the second-order phase transition at $E(5)$.

such a picture. For $N \sim 84$ there is a subshell gap in the proton single-particle energies at $Z = 64$, due to the relatively high energy of the $\pi 1h_{11/2}$ orbit. However, as valence neutrons are added, and soon begin to fill the highly overlapping $\nu 1h_{9/2}$ orbit, the attractive monopole (and quadrupole) p-n interactions [13] act to lower the energy of the $\pi 1h_{11/2}$ orbit, eventually (by $N \sim 90$) eradicating the $Z = 64$ subshell gap [14]. When this gap is still effective, the equilibrium configuration develops modest collectivity from correlations involving the proton $\pi 2d_{5/2}$ and $\pi 1g_{7/2}$ orbits below $Z = 64$. At the same time a deformed configuration builds up involving the $\pi 1h_{11/2}$ orbit. However, due to its rather high energy, this configuration is only a metastable (excited) minimum, not a global one. The energy appears as in case 1 on the left in fig. 2. As the $\pi 1h_{11/2}$ energy is lowered with the addition of valence neutrons, so is the energy of the deformed configuration [12–14]. The phase transition occurs at the critical point at which the spherical and deformed

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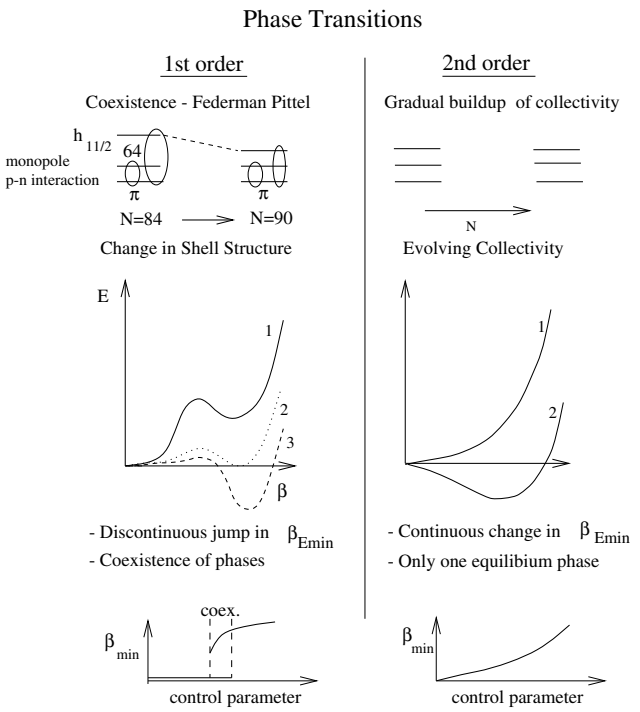

Fig. 2. Illustration of the difference between first- and second-order phase transitions and the Federman-Pittel mechanism involved in a first-order phase transition. See text.

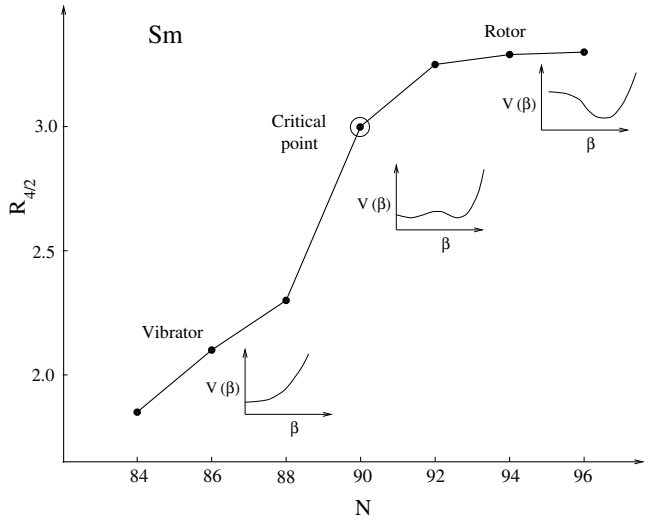

Fig. 3. $R_{4/2}$ values in Sm as a function of neutron number, with sketches of the potential for several points. Based on ref. [7].

minima are equal in energy (case 2, fig. 2, left). With still more valence neutrons the deformed solution becomes the global minimum (case 3). This scenario is known at the Federman-Pittel mechanism [12] first elucidated in the $A \sim 100$ region [12] and later extended to the $A \sim 150$ region [14]. Note that, in a first-order phase transition, there is a kink (discontinuous first derivative) in the equilibrium energy and a discontinuity in the deformation β.

It is worthwhile noting that the underlying idea of the disappearance of shell or subshell gaps, that is, the disappearance of certain magic numbers, was revolutionary in the late 1970's, but has now, in the era of exotic nuclei, become a common mantra: its origins [12–14] should not be forgotten.

The empirical systematics of the Sm isotopes show the behavior (see fig. 3) of a typical observable, $R_{4/2} \equiv E(4_1^+)/E(2_1^+)$ in such a transition region. $R_{4/2}$ values below 2.0 are characteristic of seniority-two configurations with a short-range residual interaction. Values around 2-2.3 are typical of spherical (anharmonic) vibrators while, deformed rotors give $R_{4/2} \sim 3.33$. Values near 3.0 mark the transition from spherical to deformed. The rapid rise of $R_{4/2}$ near $N = 90$ is suggestive of the rapid structural change near a phase transition. (We will see at the end that such evidence, while historically valuable, can be misleading.) The sketches of the potential against β for $R \sim 2$, 3.0, and 3.33 indicate the evolution of the equilibrium phases. At the phase transitional point (which we call, somewhat inaccurately, the "critical point") two degenerate minima appear in the potential.

2 Critical-point symmetries

It is at this point that the concept of critical-point symmetry enters. In ref. [1], we had speculated that the extreme behavior of certain observables hinted at an (unknown) underlying symmetry. Iachello recognized that, with certain approximations, a potential similar to that illustrated for the critical point in fig. 3 could be solved analytically, thus realizing that speculation in the form of the $X(5)$ symmetry. His ansatz was to simulate the rise in $V(\beta)$ at large β by a vertical (infinite) wall and to ignore the small barrier between the minima in $V(\beta)$ at intermediate β. (Both of these approximations have been assessed [15,16] and, thus far, bear out the essential predictions of $X(5)$.) Such an infinite square well in β within the context of a 5-dimensional space (β, γ and the three Euler angles) leads to a Bessel equation for the β degree of freedom, whose eigenfunctions $\Phi(\beta)$ are Bessel functions of irrational order and whose eigenvalues are the zeros of these Bessel functions.

The upshot of this solution is the level scheme for $X(5)$ shown on the left in fig. 4. A more complete level scheme is shown in ref. [5]. We stress that, except for scale, this scheme is parameter free. It exhibits a number of key predictions, namely $R_{4/2} = 3.01$, $R_{0/2} = 5.67$, and $R(6_1^+)/R(0_2^+) = 0.96$. Moreover, an approximate quantum number, s, appears, labeling different major families of states and providing approximate transition selection rules. One feature of particular note is that the $B(E2 : 2_2^+ \rightarrow 0_2^+)$ value is about 25% less than the $B(E2 : 2_1^+ \rightarrow 0_1^+)$ value. The full set of $X(5)$ characteristics constitutes a rich testing ground for the appropriate transitional nuclei.

Figure 4 includes a comparison of $X(5)$ with the data for ^{152}Sm. The empirical level scheme represents a substantial departure from previous schemes in the literature, due to new experiments, reported in refs. [1–3]

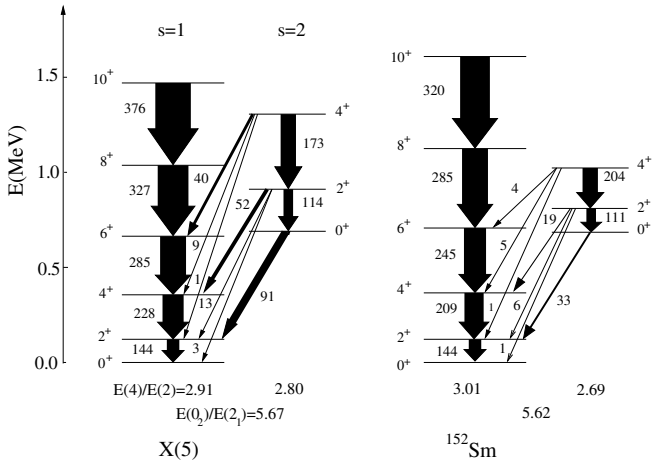

Fig. 4. Comparison of $X(5)$ and ^{152}Sm. Based on ref. [7].

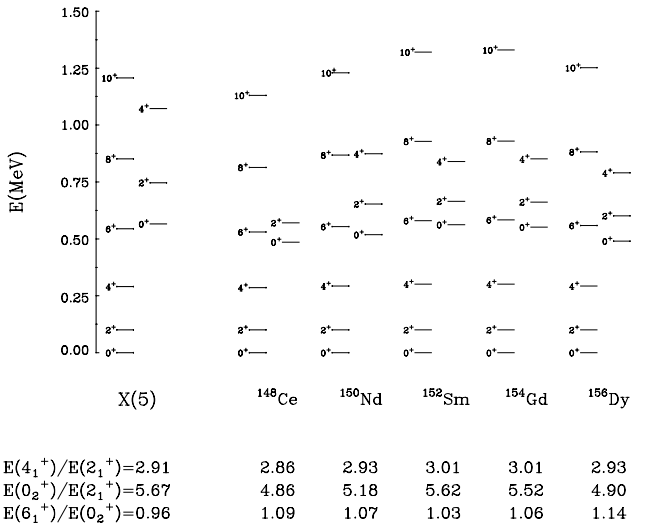

Fig. 5. Comparison of $X(5)$ and the $N = 90$ nuclei. Three key signatures of $X(5)$, and their empirical values in these nuclei, are shown at the bottom.

especially, that greatly revised certain $E2$ branching ratios, changed certain γ-ray transition multipolarities, and altered two key $B(E2)$ values [$B(E2 : 4_2^+ \rightarrow 2_2^+)$ and $B(E2 : 2_2^+ \rightarrow 0_2^+)$] in major ways. These experiments were carried out primarily at Yale and in Köln, using β-decay with clover detectors, $\gamma\gamma$ coincidence work and lifetime measurements with the Köln and Yale plunger systems. Subsequent GRID lifetime measurements [17] at the ILL also have provided new $B(E2)$ values in ^{152}Sm and neighboring nuclei.

The agreement in fig. 4 is excellent, especially considering the parameter-free nature of the $X(5)$ predictions (except for scale). The empirical yrast energies and $B(E2)$ values are intermediate between vibrator and rotor and in good agreement with $X(5)$. $R_{4/2} = 3.01$, close to the 2.91 of $X(5)$, and $R_{0/2}$ is 5.62, almost exactly the $X(5)$ prediction of 5.67. Moreover, the $B(E2 : 2_2^+ \rightarrow 0_2^+)$, as in $X(5)$, is notably less than the corresponding yrast $B(E2)$ value. The relative intersequence $B(E2)$ values (and relative $E0$ strengths —not shown) also are well reproduced by $X(5)$. Disagreements in the absolute values of these latter and in the yrare spacings have been discussed elsewhere and, while they point to deficiencies in the model, are not considered serious. In particular, the strengths of the $\Delta s = 1$ transitions can be easily reduced by a slight deviation from $X(5)$ in the direction of the rotor.

Originally, we thought that ^{152}Sm was unique in exhibiting $X(5)$ but, spurred by a question from Mark Riley, we now believe that the entire $N = 90$ region exhibits behavior quite close to $X(5)$. This is suggested by the data in fig. 5, and the values of the key $X(5)$ signatures given below the level schemes. In fact, recent lifetime data on ^{150}Nd [8] shows that this nucleus is also an excellent candidate for an empirical manifestation of the $X(5)$ symmetry —better in some respects than ^{152}Sm and less good in others. Very recent data, presented at this conference by Dewald [18], shows that ^{154}Gd is also very close to $X(5)$.

The addition of $E(5)$ and $X(5)$ to the litany of analytic paradigms of nuclear structure (see fig. 1) greatly augments our ability to analyze and interpret the structure of many nuclei.

3 The low-energy nuclear phase diagram

The symmetry triangle of fig. 1 exhibits a line of first-order phase transitions from the $X(5)$ region along the bottom leg, terminating at the second-order phase transition $E(5)$. The absence of a phase transition along the $SU(3)$-$O(6)$ (prolate rotor to γ-soft rotor) is conspicuous, although well known [19,20] for over 20 years. Nevertheless, this asymmetry is somehow viscerally disturbing and has recently prompted an extended study of structural evolution into the realm of oblate nuclei that uses the IBA to span the triangle [21]. Figure 6 (top) schematically shows the behavior of $R_{4/2}$ across the bottom leg of the symmetry triangle of fig. 1. This would be calculated using the IBA in terms of a parameter ζ that goes from 0 in $U(5)$ to 1 in $SU(3)$. The behavior is characteristic of a phase transition with a sharp rise and a change in sign of the second derivative. The point of steepest ascent, where the first derivative maximizes, is the phase transition point. The second panel of fig. 6 illustrates how $R_{4/2}$ varies along the $SU(3)$ to $O(6)$ leg of the symmetry triangle, in terms of a parameter, χ, that ranges from $-\sqrt{7}/2$ in $SU(3)$ to 0 in $O(6)$. The behavior is now qualitatively different. Though there is a sharp drop, there is no extremum in $\partial R_{4/2}/\partial\chi$ and no turning point where the second derivative changes sign.

However, and this is the key point, if the symmetry triangle is extended [21] to oblate deformations by allowing χ to continue from 0 to $+\sqrt{7}/2$ we get the results in the third panel. Now, all the classic characteristics of phase transitional behavior appear, at $\chi = 0$.

This leads to the concept of the extended symmetry triangle [21] in the lowest panel of fig. 6, where $O(6)$ is now seen to be, not only a dynamical symmetry of the IBA, but also a critical point in the phase transition from a prolate to an oblate rotor.

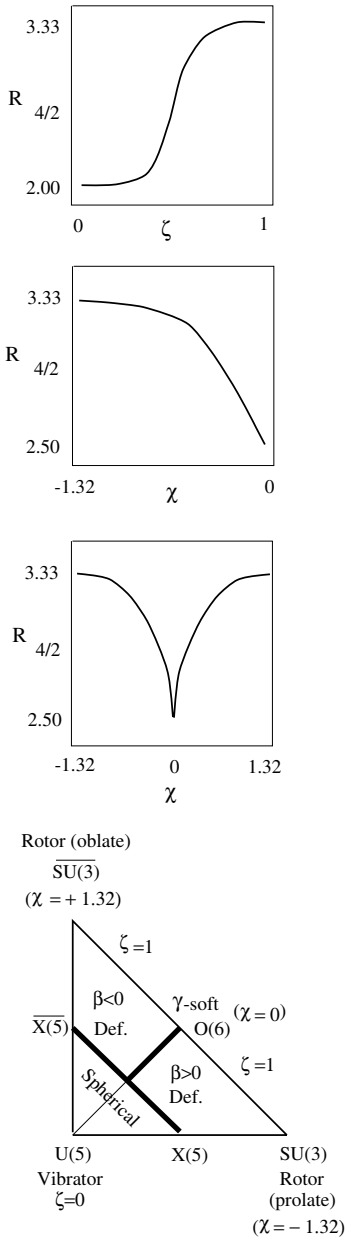

Fig. 6. Schematic illustration of the variations of $R_{4/2}$ for the vibrator to prolate rotor (top), prolate rotor to γ soft rotor (second panel), and prolate rotor to oblate rotor (third panel) as a function of the IBA parameters ζ and χ (see text). The bottom panel shows the extended symmetry triangle obtained both by numerical calculations and Landau theory. The thick black lines are lines of first-order phase transitions which meet at a point of second-order phase transition which Iachello simulated with the symmetry $E(5)$.

4 Landau theory for the nuclear equilibrium phase diagram

It is possible to achieve the same results in an extremely simple and elegant way using the essence of Landau's 1937 theory [22] of phase transitions, applying it (with the obvious caveats about finite systems and quantum fluctuations) to the nuclear case. Landau theory has previously been applied to hot, rotating nuclei (see, *e.g.*, ref. [23]). Our treatment here applies to the equilibrium low energy, low spin states of nuclei. We give a brief treatment here following the ideas originally presented in ref. [24], but casting them in a simpler and more general context.

We start with the idea of an energy functional for the nucleus given by

$$\Phi = \Phi_0 + \delta\Phi_0\,, \tag{1}$$

where Φ_0 has a higher symmetry (spherically symmetric) and $\delta\Phi_0$ breaks spherical symmetry giving a lower (deformed) symmetry. We now assume that Φ can be expanded in powers of β and γ as

$$\Phi(P,T) = \Phi_0 + A\beta^2 + B\beta^3\cos 3\gamma + C\beta^4 + \ldots\,. \tag{2}$$

In Landau theory Φ_0, A, B, and C would be functions of pressure, P, and temperature, T. To maintain the historical link to his work we will continue to use these labels although, in the nuclear case, they would be something like neutron and proton number or any two convenient model parameters, such as ζ or χ discussed above.

We note that everything that follows, including the derivation of the full nuclear equilibrium phase diagram, depends solely on the applicability of the expansion in eq. (2): that is, it is a very general treatment, free from details of specific potentials.

The essential step is to realize that, for Φ to represent an equilibrium state of the nucleus, it must be a minimum. That is, for each value of P, T, Φ must be a minimum at the value of β describing the system. (For our present purposes we will take oblate deformation to correspond to $\gamma = 0°$, $\beta < 0$ rather than the Lund convention sometimes used. This allows us to ignore derivatives with respect to the factor $\cos 3\gamma$.) Thus, if β_0 represents the equilibrium deformation, we must have

$$\left.\frac{\partial\Phi}{\partial\beta}\right|_{\beta_0} = 0 \quad \text{and} \quad \left.\frac{\partial^2\Phi}{\partial\beta^2}\right|_{\beta_0} > 0\,. \tag{3}$$

Equation (3), evaluated for Φ in eq. (2), gives

$$\beta_0(2A + 3B\beta_0\cos 3\gamma + 4C\beta_0^2) = 0\,, \tag{4}$$

$$2A + 6B\beta_0\cos 3\gamma + 12C\beta_0^2) > 0\,. \tag{5}$$

Equation (4) has two solutions:

$$\beta_0 = 0 \qquad \text{and} \qquad \beta_0 \neq 0\,. \tag{6}$$

For $\beta_0 = 0$ it is clear from eq. (5) that $A > 0$. This is the potential sketched near $R_{4/2} = 2$ in fig. 3.

For $\beta_0 \neq 0$, we first consider the case where $B = 0$. Then eq. (4) gives

$$(2A + 4C\beta_0^2) = 0$$

or

$$\beta_0 = \pm\sqrt{\frac{-A}{2C}}\,. \tag{7}$$

Of course, to avoid catastrophic asymptotics, C must be positive. Hence, for β_0 to be real A must be < 0. This corresponds to the sketch of the potential near $R_{4/2} \sim 3.33$ in fig. 3. If $A > 0$ for the higher (spherical) symmetry and $A < 0$ for the lower (deformed) symmetry, then, clearly, $A = 0$ for the phase transition. Since this is a single condition in a two-dimensional (PT plane) diagram, the condition $A = 0$ specifies a curve in this plane. The condition $B = 0$ specifies another curve, lying on the $A < 0$ (deformed) side of this plane. The two solutions for finite β_0, for $B = 0$, $\beta_0 = +\sqrt{\frac{-A}{2C}}$ and $\beta_0 = -\sqrt{\frac{-A}{2C}}$, correspond to solutions on prolate and oblate sides of the $B = 0$, $A < 0$ curve. This is illustrated in the lowest panel of fig. 6. We see that β changes discontinuously across $A = 0$, making the $A = 0$ curve a locus of a first-order phase transition, and β also jumps discontinuously across the $B = 0$ curve which must therefore also be a locus of another first-order phase transition.

The extended symmetry triangle, at the bottom of fig. 6, shows 3 lines of first-order phase transition (spherical-prolate, spherical-oblate, and prolate to oblate), and four dynamical symmetries ($U(5)$, $SU(3)$, $O(6)$, and $\overline{SU(3)}$ to use the language of the IBA). $O(6)$ is both a dynamical symmetry and a critical point. And $E(5)$ (not labeled in the figure to reduce clutter) simulates the point at the junction of the two first-order phase transitions in fig. 6. This junction is now seen to be an isolated point of second-order phase transition, corresponding to $A = 0$, $B = 0$, and is therefore identifiable as a nuclear triple point.

Finally, we see that, except for the γ-soft trajectory through this triple point, it is impossible to change symmetries without crossing a line of first-order phase transition. This suggests that phase transitional behavior may be much more widespread in nuclei than heretofore thought. This raises an interesting point. Phase transitions are, by definition, rapid. But they are rapid in terms of some control parameter (*e.g.*, ζ or χ). They are not necessarily rapid in terms of neutron or proton number. That is, the dependence of ζ and χ on N and Z may be highly non-linear. Mapping out this dependence could shed light on the relation of rapid and gradual shape transitions in nuclei.

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